

Ann and Makiko like to swim laps at the community center pool. They are swimming together today, but they are on different swim schedules. Ann swims every 3 days and Makiko swims every 5 days. How many times will they both be at the pool on the same day during the next ten weeks?

- FIND OUT What is the question you have to answer?
 - · Are Makiko and Ann swimming laps together today? Do they always swim together?
 - What is Ann's schedule?
 - What is Makiko's schedule?

CHOOSE A STRATEGY

- What will be the next day that Ann is at the pool? What will be the next day that Makiko is at the pool?
- · Is there a way to organize and lay out the information, to see which days Ann and Makiko are at the pool?

- **SOLVE IT** When you make a table, what are you keeping track of in the first row?
 - What are you keeping track of in the second row?
 - What are you keeping track of in the third row?
 - After day 3, what is the next day that Ann is at the pool? What is the next day that Makiko is at the pool after day 5?
 - Do you need to add more days to the table?
 - On which day are Ann and Makiko at the pool together again?
 - How many days from this day will they be at the pool again on the same day? Do you see a pattern?
 - How many days are there in ten weeks?
 - How many times will this pattern repeat during ten weeks?
 - How many times will they both be at the pool on the same day during the next ten weeks?

	Day	1	2	3	4	5	6	7	8	9	10	
ļ	Ann	 		X			X			, <u> </u>		
	Makiko					×						
	•						<u></u>					



It's here — the hottest item of the season, Rodney the Talking Robot! Rodney can keep anyone company, including the dog or cat. When the robots are made at the factory, they move along on a conveyor belt and every 8th robot gets a blue control panel, every 3rd robot gets blinking green eyes, and every 4th robot gets a square head. If 150 robots come off the conveyor belt, how many will have all three: a blue control panel, blinking green eyes, and a square head?

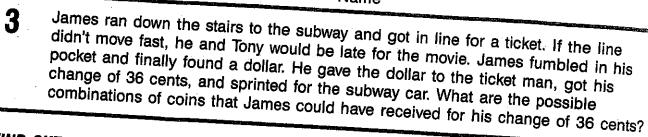
- FIND OUT What is the question you have to answer?
 - What is happening to the robots as they move along the conveyor belt? Does every robot get one of the three things on it?
 - Which robots get blinking green eyes?
 - Which robots get a blue control panel?
 - Which robots get a square head?

CHOOSE A STRATEGY

- Does the first robot get a blue control panel, blinking green eyes, or a square head? Does the second robot? Does the third robot?
- Is there a way to organize and lay out the information, so you can "see" which robots get a blue control panel, blinking green eyes, or a square head?

- SOLVE IT When you make a table, what are you keeping track of in the first row?
 - What are you keeping track of in the second row?
 - What are you keeping track of in the third row?
 - What are you keeping track of in the fourth row?
 - If you look at the table, which is the first robot with a blue control panel? Which robot has the next blue control panel?
 - · Which is the first robot with blinking green eyes? Which is the next robot with green eyes?
 - · Which is the first robot with a square head? Which is the next robot with a square head?
 - Do you need to continue the table?
 - · Which robot is the first to have all three: a blue control panel, blinking green eyes, and a square head?
 - Which robot will be the next one to have all three? Do you see a pattern?
 - How many times will the pattern repeat for 150 robots?
 - How many robots out of 150 will have a blue control panel, blinking green eyes, and a square head?

Robot	1	2	3	4	5	6	
Control panel							
Green eyes			×			×	
Square head				×	-		



- FIND OUT What is the question you have to answer?
 - What did James give the ticket man?
 - What did the ticket man give to James?

CHOOSE A STRATEGY

- What is one possible combination of coins that makes 36 cents? Can you
- How can you systematically record all the possible combinations of coins

- SOLVE IT What are the different kinds of coins that could have been in the change
 - Make a list. What do you want to keep track of in the first column of your list? second column? third column? fourth column?
 - Begin the list with 25 cents. In the first row of the list, 1 quarter is combined with 1 dime and 1 penny to make 36 cents. Is there any other way to combine 1 quarter with other coins to make 36 cents?
 - Finish filling in the possible ways to combine a quarter with other coins to make 36 cents. Then move to the dimes. What is the largest number of dimes he could have received?
 - Finish your list. What are the possible combinations of coins that James could have received for his change of 36 cents?

25¢	10¢	5¢	1¢
1	1	0	1
l	1	1	

Andre's Blue Ribbon Cars & Trucks is having a big sale. Mike and Don are setting up the lot. The boss gives them a diagram of the lot and these directions: "Put the 4-door car in front of the van. Put the jeep between the truck and the van. Put the sportscar to the left of the 2-door and 4-door cars." How did Mike and Don set up the lot?

- FIND OUT What is the question you have to answer?
 - How many cars and trucks are they setting up? What are they?
 - What do you know about the location of the 4-door car?
 - What do you know about the location of the jeep?
 - What do you know about the location of the sportscar?

CHOOSE A STRATEGY

 Would it help to have pieces of paper, or something to represent each car or truck, and be able to move them around?

- SOLVE IT If you use pieces of paper, how would you label them?
 - What goes in front of the van?
 - What can you put on either side of the jeep?
 - Where does the sportscar go?
 - Now can you find a place for the 2-door car?
 - Is there more than one solution for this problem?

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LOOK BACK

 Read the problem again. Look at the data, conditions, and the main question. Review your set-up of the lot. Is your answer reasonable?



Dinner is over, and it is the twins' turn to clean up and do the dishes. Jaime grabs the dishtowel and waits for Josh to start washing the dishes. Instead of fighting about who has to wash, Josh suggests a game to settle the dispute. He hands Jaime a die whose six faces are marked 1, 2, 3, 4, 5, and 6. To play the game, Josh explains, they take turns rolling the die 3 times in a row. The first one to roll a total of 12 in 3 rolls gets to dry tonight. How many different ways can the twins combine 3 rolls of the die to total 12?

- FIND OUT What is the question you have to answer?
 - How many dice do they have?
 - How is the die marked?
 - How many rolls does each player get?
 - What are the possible scores for one roll?
 - What is the winning total for 3 rolls?

STRATEGY

- CHOOSE A If Jaime gets 2 on the first roll, 5 on the second roll, and 5 on the third roll, is it the same as getting 5 on the first roll, 2 on the second roll, and 5 on the third roll? Is there another way Jaime can score 12 points with the numbers 2, 5, and 5?
 - · Is there a systematic way to record all the possible ways Jaime and Josh can roll 12 points?

- **SOLVE IT** What are the possible scores for one roll?
 - What are the possible combinations of numbers that equal 12 points?
 - One way to set up an organized list is to have three columns labeled Roll 1, Roll 2, and Roll 3. Begin with 1 for Roll 1, 5 for Roll 2, and 6 for Roll 3. Is there another way to arrange these numbers, using 1 for Roll 1?
 - Now use 5 for 1. How many ways can you list these same numbers, using 5 for Roll 1?
 - Now use 6 for Roll 1, 5 for Roll 2, and 1 for Roll 3. How many different ways can you list these three numbers?
 - Finish making your list. How many ways can the players score 12 points?

Combinations of 12 1 + 5 + 6	Roll 1	Roll 2	Roll 3
2 + 5 + 5	1	5	6
	1	6	5
	5	1	6

On July 5, in the area around Center Village, there was great excitement. Six different people reported to the police that they had seen Bigfoot, the large hairy creature sometimes seen but never captured. The next day, twice as many people called the police, sure they had seen the creature. Each day the police received twice as many calls as the day before. After they got a total of more than 300 calls, the police took the phone off the hook! On what day did the police receive their 300th call?

- FIND OUT What is the question you have to answer?
 - How many people first called the police?
 - How did the number of calls change from the first day to the second day?
 - Did the rate of increase stay the same from day to day?
 - When did the police take the phone off the hook?

CHOOSE A STRATEGY

- How did the calls increase each day?
- Can you use this rate of increase to help you solve the problem?
- Is there a systematic way to record the information?

- SOLVE IT If you set up a table, what are you keeping track of in the top row?
 - What are you keeping track of in the second row?
 - How many calls did the police get on day 1?
 - If the number of calls doubled, how many did they get on day 2?
 - How many calls did the police get on day 3?
 - Continue filling in your table. On what day did the police receive their 300th call?

Day	1	2	3	4	5	
Number of calls	6	12	24			
Total	6	18				

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of increase. Is your answer reasonable?

On the ancient island of Circulus archaeologists found a series of caves. In the first cave they found a circle with 560 stones. In the second cave they found a circle of stones with 8 fewer stones than in the first cave. Each new cave had a circle with twice as many stones missing from the circle than in the previous cave. For example, in the third cave there were 16 fewer stones in the circle than in the second cave. If the stone circles continued in this way, which cave would have the last stone circle?

- FIND OUT What is the question you have to answer?
 - What did the archaeologists find?
 - How many stones were in the circle in the first cave?
 - What was the difference in the circles between the first and second caves?
 - What did the archaeologists notice about each new cave?
 - What was the difference in the circles between the second and third caves?

CHOOSE A STRATEGY

- How did the circles decrease from cave to cave?
- Can you use this rate of decrease to help you solve the problem?
- How can you organize and record the information?

- SOLVE IT When you set up your table, what are you keeping track of in the first row? In the second row? In the third row?
 - What is the pattern of decrease in the stone circles?
 - How many stones are in the circle in cave 1?
 - · How many fewer stones are in the circle in cave 2? How many stones are in the circle?
 - How many fewer stones are in cave 3 than in cave 2? How many stones are in the circle?
 - Continue filling in your table. What cave will have the last stone circle?

Cave	1	2	3	4	5	
Number fewer		8	16	32		
Total in circle	560	552				

LOOK BACK • Read the problem again. Look at the data, conditions, and the main question. Review your table and pattern of decrease. Is your answer reasonable?

It is 12:00 and people are lining up for the matinee at the Bijou Cinema Six. In the first five minutes (12:05), 6 people get into line. At the end of the second five minutes (12:10), there are 11 people in line. At the end of the third five minutes (12:15), there are 16 people in line. If the people keep lining up at this rate, what time will it be when there are 81 people in line?

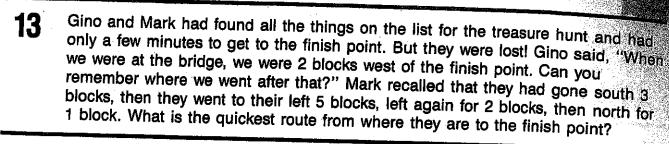
- FIND OUT What is the question you have to answer?
 - What time do people start lining up?
 - · How many people line up in the first five minutes? In the second five minutes? In the third five minutes?

CHOOSE A STRATEGY

- Would it help to keep track of every five minutes and how many people are
- What strategy can help you figure out how the line is increasing?

- SOLVE IT When you set up your table, what are you keeping track of in the first row?
 - What are you keeping track of in the second row?
 - What are you keeping track of in the third row?
 - What is the difference between the numbers of people in the first and second five minutes? The second and third five minutes?
 - Do you see a pattern of increase?
 - Continue to fill in your table. What time will it be when 81 people are in line?

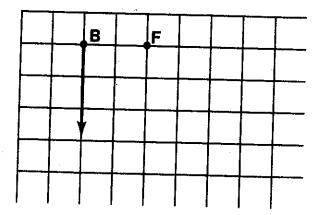
	1	2	3	4	5		
Time	12:05	12:10	12:15				
Total in line	6	11	16				



- FIND OUT What is the question you have to answer?
 - What are Gino and Mark doing? What happened to them?
 - What do Gino and Mark know about the location of the finish point?
 - What did Mark remember about where they had gone after they left the bridge?

STRATEGY

- CHOOSE A Would it help to draw a diagram, to see where Gino and Mark have gone?
- SOLVE IT Begin your map at the bridge. What can you mark on the map to the east of the bridge?
 - Where did they go first from the bridge?
 - What did Mark remember about where they went next?
 - Where did they go after that?
 - Finish mapping all the information you have. Where are they now, when they think they are lost?
 - What is the fastest route to the finish point?





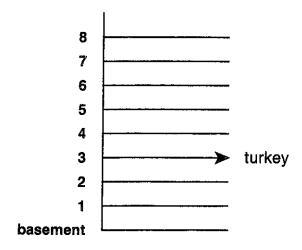
Darryl has a summer job with real ups and downs: he delivers packages in the Sears Tower in Chicago. On his first day, he is given a box of sandwiches to deliver during lunchtime. He takes the turkey sandwich to an office 3 floors above the basement. He delivers the ham and cheese 13 floors above the turkey and 7 floors below the egg salad. He delivers the pastrami 9 floors below the ham and cheese and 8 floors below the submarine sandwich. He takes the tuna to an office 12 floors below the submarine sandwich. What is the number of the floor to which Darryl delivered each sandwich?

- FIND OUT What is the question you have to answer?
 - What is Darryl doing?
 - What were the sandwiches that Darryl delivered?
 - Where does Darryl take the turkey sandwich?
 - Where does he deliver the ham and cheese?
 - Where does he deliver the pastrami?
 - Where does he deliver the tuna?

CHOOSE A STRATEGY

 Would it help to draw a picture of the building, so that you can see where Darryl is going to make his deliveries?

- SOLVE IT When you make a diagram of the building, what is the first place to mark? What else do you want to label?
 - Where does Darryl go to deliver the turkey sandwich? Which floor is that?
 - Where does he go to deliver the ham and cheese? Which floor is that on?
 - What do you know about where the egg salad goes? Which floor is that?
 - Where does he go to deliver the pastrami? Which floor is that on?
 - What do you know about where the submarine goes? Which floor is that?
 - Where does the tuna go? Which floor?
 - Can you see where each sandwich is now?





How many minutes did Heidi, Saul, and Joy each travel to get to the skating rink on Saturday? Joy came by skateboard, Heidi came by bike, and Saul came on the bus. It took Heidi twice as long as Joy to get there. It took Saul 10 minutes more than it took both the girls together. All three skaters together took 64 minutes to get to the rink.

- FIND OUT What is the question you have to answer?
 - What do you know about Heidi's traveling time?
 - What do you know about Joy's traveling time?
 - What do you know about Saul's traveling time?
 - How long did it take all three skaters together to get to the rink?

- CHOOSE A Will guessing an answer help you to solve this problem?
- **STRATEGY** How can you use the information from an incorrect guess?

- SOLVE IT Would you start by making a guess for Heidi, Saul, or Joy? Why?
 - What is your guess?
 - If you make a guess for one skater, can you figure out the time taken by the other two skaters?
 - How can you check your guess?
 - · How was your guess? If your first guess was wrong, how can you use the information to make your next guess?

LOOK BACK



This notice was posted: Auditions Next Week for the School Play. After reading a copy of the play, Betty, Hilda, Adam, and Renee have decided which parts to try out for. Hilda wants a part that has twice as many lines as the part that Betty wants. Adam has chosen a part with three more than twice as many lines as the part that Hilda wants. Renee is going to try out for a part with four more lines than Betty's part. If they get the parts they want, together they will have a total of 47 speaking lines. How many lines would each actor have in the play?

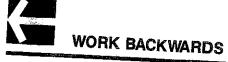
- FIND OUT What is the question you have to answer?
 - Who is trying out for the school play?
 - What do you know about Hilda's part?
 - What do you know about Adam's part?
 - What do you know about Renee's part?
 - What do you know about Betty's part?
 - What is the total number of speaking lines for the four parts that Hilda, Adam, Renee, and Betty want?

CHOOSE A

- Will guessing an answer help you to solve this problem?
- STRATEGY How can information from an incorrect guess help you?

SOLVE IT

- Would you begin with a guess for Hilda, Adam, Renee, or Betty? Why?
- What is your guess?
- If you make a guess for one of the actors, can you figure out the number of lines for the other actors?
- How can you check your guess?
- How was your guess? If your first guess was wrong, how can you use the information to make your next guess?



Kevin, Barbara, and their mother and father went backpacking in Yosemite National Park. On the first and second days, each hiker had a serving of food for breakfast, lunch, and dinner. A large, noisy, brown bear barged into camp the second night, got the food pack down from the tree where they had hung it, and ate one half of the food that was left. The next morning, after they all had breakfast, they found they had 4 food servings left. They decided they had better hike back to their car. How many servings of food did they begin the trip with?

- FIND OUT What is the question you have to answer?
 - How many hikers were eating at each meal?
 - What do you know about the number of servings eaten on day 1 and 2? How much did the bear eat?

 - How many servings did the hikers have after the bear had his share?
 - How many servings of food were left at the end of the hike?

CHOOSE A STRATEGY

- Think about the best way to begin solving this problem. The only specific information you have is how much food is left on the last day. Therefore you need to work backwards from the last day to the first day.
- Is there a systematic way to record the information?

- SOLVE IT If you begin with the last day of the trip, how many servings of food did
 - How much did the hikers eat on the last day? How many servings did the hikers have altogether before breakfast the last day?
 - Can you figure out how much the bear ate?
 - How many servings of food were left at the end of the second day before
 - How many servings were eaten on the second day?
 - How many servings were eaten on the first day?
 - How many servings did they begin with?

Dav 3 - 4 left 4 eaten

Day 2 -

Keri's father, Bill, was a baker. Out of his usual morning batch of chocolate chip cookies, Bill burned the first two dozen cookies. He gave half of what was left to Keri to take to school. He wrapped up half of the remaining cookies and gave them to the gas station crew next door. He gave half of what was left to the policeman on the beat. If Bill had only 7 cookies left, how many cookies were in the batch of chocolate chip cookies?

- FIND OUT What is the question you have to answer?
 - What was Bill doing?
 - What was the first thing that happened to Bill's cookies?
 - How many cookies did Bill give Keri?
 - How many cookies did Bill give the gas station crew?
 - How many cookies did Bill give to the policeman?
 - How many cookies did Bill have left?

CHOOSE A STRATEGY

- You have very little specific information. If you begin with the information you have, then what can you do?
- Is there a way to show your work that will help?

- SOLVE IT Begin with what Bill had left after he burned and gave away his cookies: 7 cookies. Who is the last person(s) that Bill gave cookies to? How many?
 - If you continue to work backwards, who is the next to last person(s) that he gave cookies to? How many?
 - Who is the other person(s) that Bill gave cookies to? How many?
 - What is the first thing that happened to the cookies?
 - Now can you figure out how many cookies were in the first batch?

$\frac{1}{2}$ of what is left	$\frac{1}{2}$ of wh	nat is left
	1/2	7

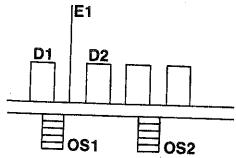
19 Sandy is delivering pizza to the second floor of the apartment house at 645 Birch Street. There are two outside stairways from the street to the building. Then there are four doors into the building. Once inside there are two elevators and two inside stairways that go to the second floor. How many different ways can Sandy go from the street to the second floor and deliver the pizza?

- FIND OUT What is the question you have to answer?
 - Where is Sandy going?
 - How many stairways go from the street to the building?
 - How many doors are there into the building?
 - How many elevators are there?
 - How many inside stairways are there?

CHOOSE A STRATEGY

• It is difficult to "see" this problem in your mind. Is there a good way to lay out the information?

- SOLVE IT Begin by showing the outside stairways up to the building. How many are there? Then draw the doors into the building. How many are there? How many elevators do you need to show that go to the second floor? How many stairways go to the second floor?
 - Begin with one outside stairway. If you can go up one stairway, through 4 doors and up 2 elevators, how many different ways can you go? Now if you go up the same stairway, through 4 doors, and up 2 inside stairways, how many different ways can you go? How many ways are there altogether from one outside stairway?
 - Do the same thing with the other outside stairway. How many different ways can you go from this stairway to the second floor?
 - How many different ways can Sandy go from the street to the second floor and deliver the pizza?



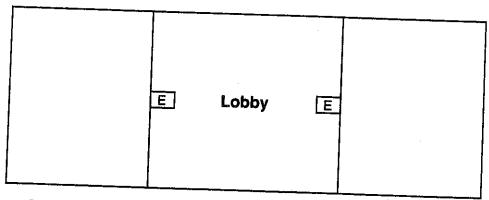
Being a double agent, Mr. X must be very careful about his daily routine to insure 20 his safety. He never eats at a restaurant more than once, and he must be careful going back and forth to the hotel room where he is presently hiding. There are four street entrances to the hotel: two entrances into the lobby, one entrance in the east wing, and one entrance in the west wing. There are six elevators in the building: two in the lobby and two in each wing. The elevators only go as high as the 15th floor. Each wing also has a stairway which goes to every floor of the hotel. How many different ways can Mr. X get to his room by entering the hotel, taking the elevator to the 15th floor, and walking up to his floor?

- FIND OUT What is the question you have to answer?
 - Where is Mr. X going?
 - How many entrances to the hotel are there?
 - How many elevators to the 15th floor are there?
 - How many stairways up to Mr. X's floor are there?

CHOOSE A STRATEGY

· Would it be helpful to "see" all this information in the form of a picture, so you can get a better idea of Mr. X's choices?

- SOLVE IT If you make a diagram of the hotel, how many entrances do you need
 - How many elevators need to be included on your diagram?
 - How many stairways do you need to mark on the diagram?
 - If Mr. X begins outside the hotel, and has a choice of 4 entrances and then 6 elevators to the 15th floor, how many different ways can he get to the 15th floor? Now how many ways can he walk up to his floor from the 15th floor? Then how many ways can he get from the street to his room?





USE LOGICAL REASONING

Name

Bill, his sister Martha, and Ann are sitting down to learn a new game from Phil Hill. Each player has a partner and the partners are seated across the table from each other. Bill Dill is sitting to the right of Phil's sister. Phil Hill is sitting to the right of Bill's sister's partner. Where is each player sitting at the table?

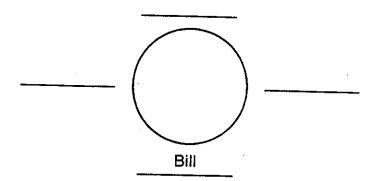
FIND OUT

- What is the question you have to answer?
- Who are the players?
- How are the players sitting at the table?
- What do you know about where Bill Dill is sitting?
- What do you know about where Phil Hill is sitting?

CHOOSE A STRATEGY

- You can use a series of "If this is true, then this must be true," statements to help you solve this problem. What kind of thinking do we call this?
- Is there a way to show your work that can be helpful?

- SOLVE 17 What are you going to put in a diagram?
 - Begin with one player. Write the name of Bill Dill on one side of the table. What do you know about where Bill is sitting?
 - If Martha is Bill's sister, then who is Phil's sister? Where is she sitting?
 - If there are two places left at the table then Phil has to be in one of them. Could Phil be to the right of Martha? Is there only one place where Phil can be sitting?
 - Where is each player sitting at the table?



Donna's father, an archaeologist, is sharing his latest diggings with Donna's class. stone tablets from an ancient culture. The number notation system used on the tablets included squares and circles. Donna holds up one stone tablet that looks like this:

The square and circle on this tablet represent the number 14. The next stone tablet looks like this: [

The squares and circles on this tablet represent the number 34. Then Donna holds up a third tablet that looks like this:

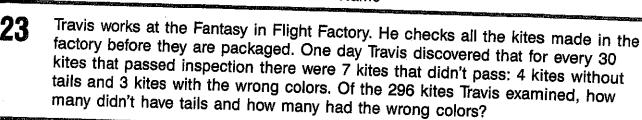
What number do the square and circles represent on the third tablet?

- FIND OUT What is the question you have to answer?
 - What are Donna and her father sharing with the class?
 - What do the squares and circles on the tablets stand for?
 - What number is represented by the square and circle on the first tablet?
 - What number is represented by the squares and circles on the second tablet?

STRATEGY

- CHOOSE A What kind of thinking can you use to organize the information in this problem?
 - Is there another strategy you can use with this problem?

- SOLVE IT What do you know about the first tablet?
 - Try guessing some numbers for the square and the circle; which numbers will add up to 14?
 - If you find two numbers that work for the first tablet, then what is the next step? Do you need to do more guessing and checking?
 - If you have values for the square and circle that give you 14 and 34 for the first and second tablets, then can you find a value for the mystery tablet?



- FIND OUT What is the question you have to answer?
 - What is Travis doing?
 - How many kites pass inspection?
 - · How many kites do not pass inspection? How many of these don't have tails? How many have the wrong colors?
 - How many kites did Travis look at?

CHOOSE A STRATEGY

- For 30 kites that passed inspection, 7 did not pass. For 60 kites that passed, how many wouldn't pass? How many would be missing tails? How many would have the wrong colors?
- How can you keep track of the kites in a systematic way?

SOLVE IT

- When you set up your table, what are you keeping track of in the first row?
- What are you keeping track of in the second row?
- What are you keeping track of in the third row?
- What are you keeping track of in the fourth row?
- · How many kites don't pass inspection for 90 kites that do? How many are missing a tail? How many have the wrong colors?
- Continue to fill in the table until you have a total of 296 kites.
- When you have a total of 296 kites, how many don't pass inspection because they are missing a tail? How many have the wrong colors?

Pass	30	60	90			
No tail	4	8				
Wrong color	3	6			<u>. </u>	
Total	37	74		<u> </u>		



The Rialto Theater is celebrating its 11th anniversary. In honor of the occasion, they are giving away free passes! They have hidden a gold, silver, purple, or green star under every seat. Every person who sits in a seat with a gold star gets a free pass to the next show. For every 2 gold stars they hid 18 silver, 16 purple, and 12 green stars. If there are 384 seats in the Rialto Theater, how many people won free passes?

- FIND OUT What is the question you have to answer?
 - What are they doing at the Rialto Theater for their 11th anniversary? How does someone get a free pass?
 - What kind of stars did they put under the seats?
 - For every 2 gold stars how many silver were there? purple? green?
 - How many seats are there in the Rialto?

CHOOSE A Strategy

- · You have to keep track of gold stars, silver stars, purple stars, and green stars. What else do you need to keep track of?
- How can you systematically record all the information you need?

SOLVE IT

- When you make a table, what are you keeping track of in the first row?
- What are you keeping track of in the second row? third row? fourth row? fifth row?
- Why do you need to keep track of the total number of stars?
- · After filling in the first column, with 2 gold stars in the first row, how many seats have stars on them?
- After filling in the column with 4 gold stars in the first row, how many seats have stars on them?
- Continue to fill in the table until you have a total of 384 stars which represent 384 seats in the theater.
- When you have stars for 384 seats, how many gold stars are hidden? How many people won free passes?

Gold	2	4	
Silver	18	36	
Purple	16	32	
Green	12		
Total	48		

26

Jessica is hiking in the mountains with her llama, her dog, and her cat. She comes to a deep river, where she must help each animal across, one at a time. She has a problem: she can't leave the llama alone with the dog, and she can't leave the cat alone with the dog. What is the fewest trips Jessica can make to get the llama, the dog, and the cat to the other side of the river?

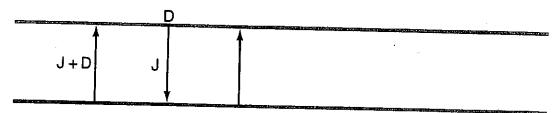
FIND OUT

- What is the question you have to answer?
- What does Jessica have to do?
- What are the conditions for crossing the river?
- What are the special conditions for the llama and the dog?
- What are the special conditions for the cat and the dog?

CHOOSE A STRATEGY

- What is the best way to keep track of all the river crossings?
- is there another strategy that might be useful here?

- **SOLVE IT** What do you need to keep track of on your diagram?
 - Who would you pick to make the first crossing? Why?
 - · If Jessica returns alone on crossing 2, who would be a good choice for crossing 3? Why?
 - What should Jessica do on crossing 4? Why?
 - If it is crossing 5 and the llama is on the other side, should Jessica take the cat or the dog? What happens if she takes the cat? What happens if she takes the dog?
 - What is the fewest trips Jessica can take to get everyone across?



Steven is leaning over the video game, concentrating on shooting down as many **28** spaceships as he can within the time limit. Vulcan spaceships are worth 20 points each, and Android spaceships are worth 25 points. If he shoots down 21 spaceships for a total of 465 points, how many Vulcan spaceships and how many Android spaceships did Steven shoot down?

- FIND OUT What is the question you have to answer?
 - What is Steven doing?
 - How many different kinds of spaceships are there?
 - How much are Vulcan spaceships worth?
 - How much are Android spaceships worth?
 - How many spaceships does Steven shoot down?
 - How many points are the spaceships worth?

CHOOSE A STRATEGY

 How can you systematically lay out the information for this problem?

- SOLVE IT How many columns do you need in your list? What are you going to put at the top of each column?
 - What are you keeping track of in each row of your list?
 - · Fill in several rows of your list. Do you have enough spaceships that you can combine numbers from each column to make 21?
 - · Fill in more rows of your list. What are some combinations of numbers that add up to 21?
 - When you find two numbers that make 21, what does the combined value of these spaceships need to be? What is the value of eleven Vulcan spaceships and ten Android spaceships?
 - Continue to fill in your list and test combinations of numbers that make 21. How many Vulcan and Android spaceships did Steven shoot down?

 Vulcan			Android				
1	20		1	25			
2	40		2	50			



At Marvelous Mark's T-Shirts, you can win a free T-shirt if you can solve the pebble puzzle. Next to the jar at the entrance is this information:

There are 825 pebbles in the jar. There are 375 more brown pebbles than white pebbles. Guess the number of brown and white pebbles in the jar and win a free T-shirt!

What is the right answer to the pebble puzzle?

- FIND OUT What is the question you have to answer?
 - What is going on at Marvelous Mark's T-Shirts?
 - How can you win a free T-shirt?
 - How many pebbles are there in the jar?
 - What do you know about the brown pebbles?

CHOOSE A STRATEGY

- Will guessing the answer help you to solve this problem?
- How can you use the information from an incorrect guess?

- **SOLVE IT** What is the total number of pebbles in the jar?
 - How many more brown pebbles than white pebbles are there?
 - Make a guess, keeping in mind the total number of pebbles and how many more brown than white there are. What is your guess?
 - How can you check your guess? How did you do?
 - . If your guess was wrong, how can you use the information to make your next guess?
 - Keep making guesses and checking them until you get the right answer. How many brown and how many white pebbles are in the jar?





At a recent convention of millipedes and centipedes held at the Hotel Fleeflea, the desk clerk registered 293 guests. He noticed that each millipede had 16 legs and that each centipede had 14 legs. Seth, the shoeshine spider, reported that there were 4,408 legs in total at the convention. He had to shine all their shoes! Can you help the desk clerk at the Hotel Fleeflea figure out how many of the registered guests were millipedes, and how many were centipedes?

Name

- FIND OUT What is the question you have to answer?
 - What was happening at the Hotel Fleeflea?
 - Who was attending the convention?
 - What did the desk clerk notice about the millipedes and centipedes?
 - What did Seth report to the desk clerk about the total number of legs at the convention?

CHOOSE A STRATEGY

- Will guessing the answer help you to solve this problem?
- How can you use the information from an incorrect guess?

SOLVE IT

- What was the total number of centipedes and millipedes?
- How many legs did each centipede have?
- How many legs did each millipede have?
- Make a guess, and remember that the total number can't be more than 293. How many centipedes? How many millipedes?
- How can you check your guess? How did you do?
- If your guess was wrong, how can you use this information to make your next guess?
- Keep making guesses and checking them until you find the right combination. How many millipedes were there? How many centipedes?



MAKE A PICTURE OR DIAGRAM

Name

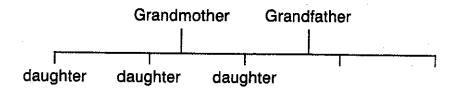
The Chen family has one grandmother and one grandfather, five mothers and five fathers. Each mother has three daughters and each daughter has two brothers. The grandmother has only one daughter-in-law. What is the fewest members of the Chen family there could be?

- FIND OUT What is the question you have to answer?
 - Who are the oldest members of the Chen family?
 - How many mothers are in the family? How many fathers?
 - How many daughters does each mother have?
 - How many brothers does each daughter have?
 - How many daughters-in-law does the grandmother have?

CHOOSE A STRATEGY

- Would it be helpful to try and "see" this problem by making a picture or diagram?
- Is there another strategy that would be helpful to use with this problem?

- **SOLVE IT** Draw a family tree diagram. Who goes at the top of the family tree?
 - How many children do the grandmother and the grandfather have? How did you come to this conclusion? Where are you going to put these children on the family tree?
 - How many of these children are married? How many daughters-in-law does the grandmother have?
 - What clue do you have to the number of grandchildren there are? How many girls? How many boys? Where are you going to put the grandchildren on the family tree?
 - What is the fewest members of the Chen family there could be?



Mario, Jake, Erica, and Erica's sister Charlene all enjoy sports. Jake likes to play tennis and always carries a racket. Mario and Erica's sister can't swim. Neither Jake nor Mario have a bike. Can you match each person with these items: a bike, a skateboard, a tennis racket, and a surfboard?

- FIND OUT What is the question you have to answer?
 - Who enjoys sports?
 - What do you know about Jake?
 - What do you know about Mario and Erica's sister?
 - What do you know about Jake and Mario?
 - · What sports equipment do you have to match up with Mario, Jake, Erica, and Charlene?

CHOOSE A STRATEGY

- The information in this problem is given in a set of clues. If you use a series of "If . . . then" statements to solve this problem, what kind of thinking will you be using?
- Is there another strategy you can use to record the information?

- SOLVE IT When you set up a table, how many columns do you need? How
 - How are you going to label the columns? The rows?
 - What do you know about Jake?
 - In which box of the table can you put a Y for yes?
 - If you match Jake with one of the items, then you can put an N in the other boxes in Jake's column. Where can you put more Ns?
 - What do you know about Mario and Erica's sister? Where can you put an N in each column for them?
 - Now you should be able to draw a logical conclusion about who belongs to the surfboard. Who has the surfboard?
 - Continue to fill in the table until all the boxes have a Y or N. Who belongs to the bike? skateboard? tennis racket? surfboard?

	Mario	Jake	Erica	Charlene
Bike				
Racket		Υ		
Skate- board				
Surf- board				

LOOK BACK ● Read the problem again. Look at the data, conditions, and the main question. Review your table and logical reasoning. Is your answer reasonable?

It is 7:00 P.M. on Back-to-School Night. Mrs. Anderson is welcoming the parents in her first class: Mr. Black, Mr. Green, Mr. White, Mrs. Brown, and Mrs. Rojo. She sees five of her students: Peter, Mary, Jack, Sam, and Jill. Mrs. Anderson notices that Mr. Green's daughter did not inherit his freckles; Mrs. Brown has big dimples when she smiles; Mary and Sam both have freckles and dimples; Mr. Black's son looks just like him; and Sam's father and Peter's father were unable to attend. Can you match up each student with a parent?

- FIND OUT What is the question you have to answer?
 - What is Mrs. Anderson doing?
 - Who are the parents and students that came to Back-to-School Night?
 - What do you know about Mr. Green? Mrs. Brown? Mary and Sam? Mr. Black? Sam and Peter?

STRATEGY

- CHOOSE A What kind of thinking can you use to organize the information in this problem?
 - Is there another strategy you can use to record the information?

- SOLVE IT When you set up your table, how many columns do you need? How many rows?
 - How are you going to label the columns? the rows?
 - What do you know about Mr. Green?
 - What do you know about Mary and Sam?
 - If Mary has freckles and Mr. Green's daughter does not, then what can you conclude about the only other girl?
 - If you can put a Y in one of the boxes, then where can you put Ns?
 - What do you know about Mr. Black?
 - What do you know about Sam and Peter?
 - If Sam's and Peter's fathers didn't come, but Mr. Black has a son, then what can you conclude about the only other boy?
 - What do you know about Mrs. Brown?
 - · Continue to use "If...then" statements and fill in the boxes in the table, until you have marked each box with a Y or N.
 - Who belongs to Mr. Black, Mr. Green, Mr. White, Mrs. Brown, and Mrs. Rojo?

	Магу	Sam	Peter	Jack	Jili
Mr. Black					
Mr. Green		<u> </u>			
Mr. White					
Mrs. Brown					
Mrs. Rojo					

Armando has a card trick for Andrew: "I have 10 cards, numbered from 1 to 10. I have arranged the cards in a stack in a special way. The first card facing up is a 1, and I'm putting it on the table. The second card I'm putting at the bottom of the stack. The third card, which is a 2, I'm putting on the table next to 1. Then the fourth card goes to the bottom of the stack. I'll continue putting one card on the table and the next card to the bottom of the stack until I put card 10 on the table. How did I first arrange the cards in the stack?"

- FIND OUT What is the question you have to answer?
 - What is Armando doing?
 - How many cards does Armando have? What are they?
 - How does Armando lay out the cards?
 - What is the first card in the stack? Where does Armando put it?
 - Where does Armando put the second card?
 - What is the third card? Where does he put it?
 - Where does he put card 10?

CHOOSE A STRATEGY

- Would it help to move around pieces of paper that represent the cards?
- Is there another strategy that can help organize your thinking about this problem?

- SOLVE IT Do you know the number of the first card in the stack? second? third? fourth? fifth?
 - Try laying out the cards like this diagram, where each X means you leave a space for a card that goes under the stack: 1 X 2 X 3 X 4 X 5 X

Be sure to do this before going to the next questions.

- When you leave a space after 5, how many cards are accounted for? Where will the 1, 2, 3, 4, and 5 go? Where are the cards that the Xs stand for?
- Can you go back and fill in the first space now?
- Do you know what goes in the space after 2, or does this card go under again?
- What goes in the space after 3?
- What do you know about the card after 4?
- What goes in the space after 5?
- Fill in all the spaces. What order do you have the cards in?

LOOK BACK

 Read the problem again. Look at the data, conditions, and the main question. Review your arrangement of cards. Is your answer reasonable?



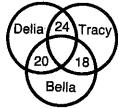
Delia, Tracy, and Bella are going cross-country skiing with a school group. They are carrying packs and each pack can weigh up to and including 10 pounds. The packs are weighed two at a time. Delia and Tracy weigh their packs together and the total is 24 pounds. When Delia and Bella weigh their packs, the total is 20 pounds. Tracy's and Bella's packs together weigh 18 pounds. Which skiers have packs that are too heavy, and by how much?

- FIND OUT What is the question you have to answer?
 - What are Delia, Tracy, and Bella doing?
 - How many pounds are allowed per pack?
 - How are the packs being weighed?
 - What is the combined weight of Tracy's and Delia's packs?
 - What is the combined weight of Delia's and Bella's packs?
 - What is the combined weight of Tracy's and Bella's packs?

CHOOSE A STRATEGY

- You can use a series of "If...then" statements to solve this problem. What kind of thinking do we call this?
- · Is there a special kind of diagram that could help you organize the information?

- SOLVE IT When you make your Venn diagram, how many intersecting circles do you need? What are you going to label each circle?
 - How are the packs weighed? Where do you put these numbers in the circles? Where does the number go for Tracy's and Delia's packs? the number for Delia's and Bella's packs? the number for Tracy's and Bella's packs?
 - There are two numbers in Delia's circle. What other number goes in the circle and where does it go?
 - If you estimate a number for Delia's pack, then you have to add it to what number to make 24? At the same time you have to add it to what number to make 20? If you estimate a number for Delia's pack, then you have to estimate what other numbers?
 - How do you go about checking your estimates? If your estimates are incorrect, how can you use this information?
 - · Now make estimates, based on the totals in the intersections. Check to see that the numbers in each two circles add up to the total in the intersection between those two circles.
 - Which packs are over the limit, and by how much?





A group of 13 friends were planning a trip. On the night before they left they made a lot of phone calls. Each friend talked to every other friend at least once. What is the fewest phone calls that could have been made?

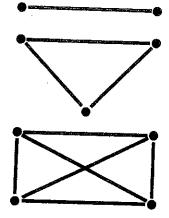
- FIND OUT What is the question you have to answer?
 - Who was planning a trip?
 - What did the friends do the night before the trip?
 - Who did each friend talk with?

CHOOSE A STRATEGY

- · Would it help to solve this same problem with a smaller number of friends, then apply what you've learned to a larger number?
- Are there other strategies that you can use with the first one?

SOLVE IT

- When you set up a table, what are you going to keep track of?
- If you begin with 2 friends and draw a diagram, how many calls will be made?
- If you have 3 friends, how many calls will be made?
- If you have 4 friends, how many calls will be made?
- How many calls will be made for 5 friends?
- Do you see a pattern in the way the number of calls increases?
- What is the fewest phone calls that could have been made?



Number of friends	Calls		
2	1		
3	3		
4	6		

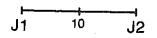
The annual Chinese New Year's parade is ready to begin. The parade marshal has stationed the 15 judges along the parade route at 10-yard intervals, to judge the students' papier-mache masks. At what place should the judges meet for ballot counting, so that together they travel the fewest possible yards?

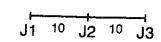
- FIND OUT What is the question you have to answer?
 - What is about to begin?
 - Who is stationed along the parade route? How many are there?
 - How far apart are the judges stationed?
 - What are the conditions for the judges' meeting place?

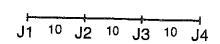
CHOOSE A

- Would it help to solve this problem first for 2 and 3 judges?
- **STRATEGY** How can you organize the information in this problem?

- SOLVE IT If you make a diagram for 2 judges, what labels do you need to put on your diagram?
 - What are the possible meeting places for 2 judges?
 - Which is the best meeting place for 2 judges?
 - If you make a diagram for 3 judges, what labels do you need?
 - What are the possible meeting points for 3 judges?
 - What is the best place for them to meet, so that together they travel the fewest possible yards?
 - Make a diagram for 4 judges. Then try making a diagram for 15 judges. What labels do you need?
 - What are the possible meeting points?
 - · Where is the best place for the judges to meet, so that together they travel the fewest possible yards?









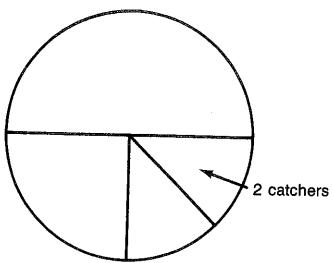
Sara's softball team can be divided into four groups: $\frac{1}{2}$ the players are strong hitters, $\frac{1}{4}$ are good pitchers, $\frac{1}{8}$ like to play the outfield, and 2 players are catchers. If each player is in only one group, how many players are on the team and how many players are in each group?

- FIND OUT What is the question you have to answer?
 - · How many groups can you divide Sara's team into? Are there any conditions given about the groups?
 - What do you know about the number of players on the team who are hitters?
 - What do you know about the number of players on the team who are pitchers?
 - What do you know about the number of players on the team who like the outfield?
 - How many players on the team are catchers?

STRATEGY

- CHOOSE A To solve this problem you need to begin with the specific information you have, the number of catchers. How can you organize the rest of the information?
 - Is there a good way to lay out the information?

- **SOLVE IT** What kind of diagram would you make?
 - How can you divide up the diagram? How can you label each part?
 - Where can you begin filling in the diagram?
 - If you work backwards, what can you fill in next?
 - Continue to work backwards and fill in each part of the diagram. How many players are on Sara's team? How many players are in each group?





MAKE IT SIMPLER

Name

Your sock drawer has 25 electric yellow socks, 30 blue striped socks, 17 orange socks, 13 magnetic magenta socks, 33 pale purple socks, 30 royal red socks, 11 gruesome green socks, 14 midnight black socks, and 23 bruin brown socks! If you reach into the drawer in the dark, how many socks do you need to pull out to be sure you have a matching pair?

- FIND OUT What is the question you have to answer?
 - How many yellow socks are in the drawer? blue? orange? magenta? purple? red? green? black? brown?
 - How many different colors are there?

CHOOSE A

- Would it be easier to solve this problem with fewer different colors?
- STRATEGY Is there another strategy you can use along with the first one?

- SOLVE IT Begin with 3 different colors and 2 socks of each color: yellow, red, and green.
 - If you pull out 1 yellow, what are the possibilities for the next sock you pull out? What are your chances that it is yellow?
 - If you pull out a green sock, you have 1 yellow and 1 green. What are the possibilities for the next sock you pull out? What are your chances that it is yellow or green?
 - If you pulled out a red sock, then you have 1 yellow, 1 red, and 1 green. What are the possibilities for the next sock? What are your chances of pulling out either red, green, or yellow?
 - Now consider the problem you started with. How many socks do you need to pull out to be sure you have a matching pair?